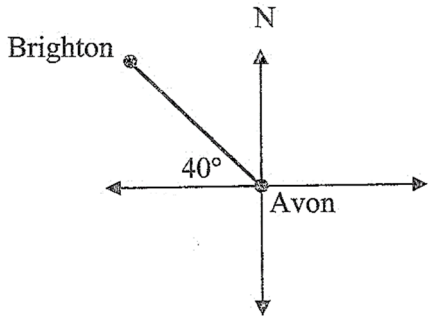
Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 10

---

1. What is the compass bearing of Avon from Brighton in the diagram shown?



A. S50°E

B. S40°W

C. N40°E

D. N50°W

2. Solve for x: $4^{x+2} \times 2^{2x-3} = 8^x$

A. $x = 2$

B. $x = -2$

C. $x = -1$

D. $x = 1$

3. Which equation represents the line parallel to $2x - 3y = 8$, passing through the point (-1,2) ?

A. $3x + 2y - 1 = 0$

B. $3x + 2y - 8 = 0$

C. $2x - 3y - 8 = 0$

D. $2x - 3y + 8 = 0$

4. A parabola is concave down and its vertex is (2,0). Which statement about the discriminant (Δ) of the parabola is correct?

A. $\Delta > 0$

B. $\Delta = 0$

C. $\Delta < 0$

D. $\Delta \leq 0$

5. A function is defined by the following rule:

$$f(x)\begin{cases} 0 & \text{if} \quad x \leq -2 \\ -1 & \text{if} \quad -2 < x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$

What is the value of $f(a^2)$?

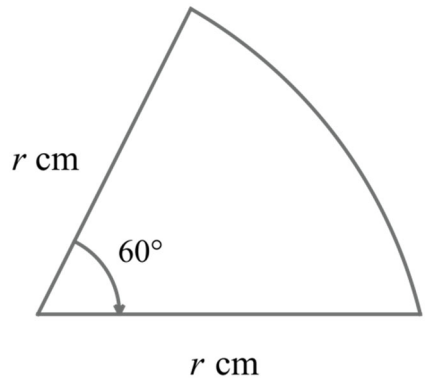
A. a^2

B. x

C. -1

D. 0

6. The sector below has an area of 10π square units. The value of r is:



A. $\sqrt{60}$

B. $\sqrt{60}\pi$

C. $\sqrt{\frac{\pi}{3}}$

D. $\sqrt{\frac{1}{3}}$

7. What is the greatest value of the function $f(x) = 3 - 2cosx$?

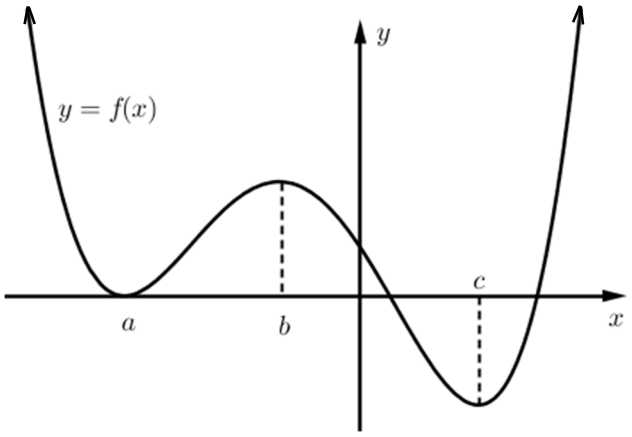
A. 1

B. 2

C. 3

D. 5

8. What is the graph of the gradient function for the curve $y = f(x)$?



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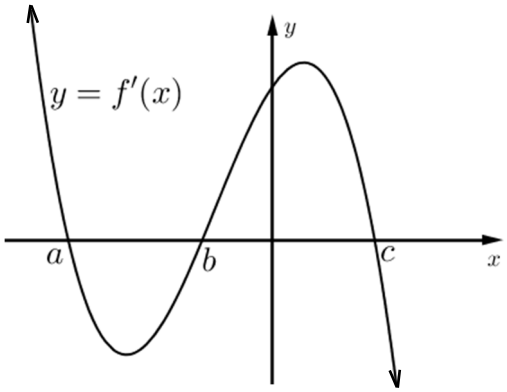
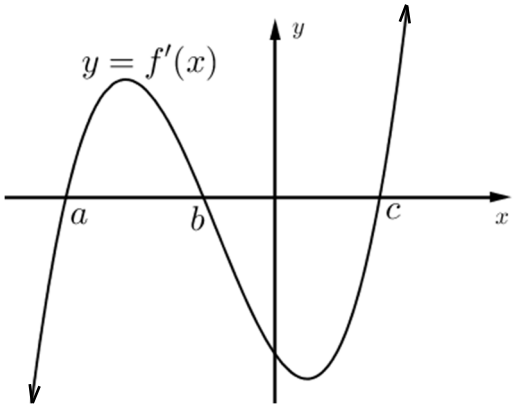
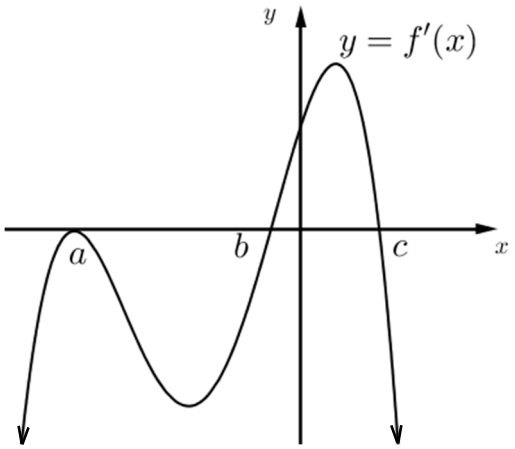


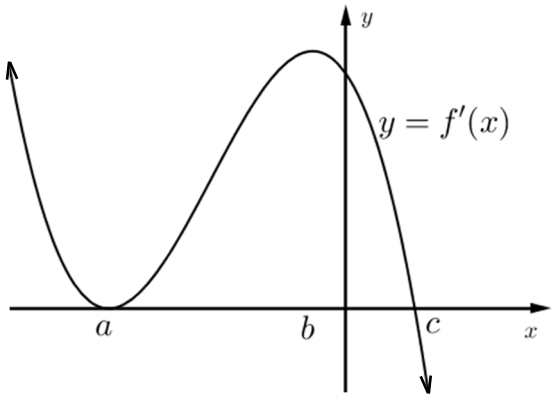
B.



C.



D.



9. Which of the following is the correct expression for differentiating $f(x) = x^2 - 2x$ with first principles?

A. $f'(x) = \lim_{h \to 0} \frac{(x-h)^2 - 2(x-h) - (x^2 - 2x)}{h}$

B. $f'(x) = \lim_{h \to 0} \frac{(x-h)^2 - (x-h) - (x^2 - 2x)}{h}$

C. $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$

D. $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + (x^2 - 2x)}{h}$

10. A particle moves in a straight line such that its distance *x* metres from the origin after *t* seconds is given by $x = t^3 - 9t^2 + 24t + 4$.
Find when and where the particle comes to rest.

A. $t = 2\ sec\ \ and\ t = 4\ \ sec,\ \ x = 20\ m\ \ \ and\ \ x = 24\ m$

B. $t = 0\ sec\ \ and\ t = 4\ \ sec,\ \ x = 20\ m\ \ \ and\ \ x = 4\ m$

C. $t = 0\ sec\ \ and\ t = 2\ \ sec,\ \ x = 24\ m\ \ \ and\ \ x = 4\ m$

D. $t = 2\ sec\ \ and\ t = 4\ \ sec,\ \ x = 20\ m\ \ \ and\ \ x = 4\ m$

Section II – 90 marks

NESA Number: | | | | | | | | |

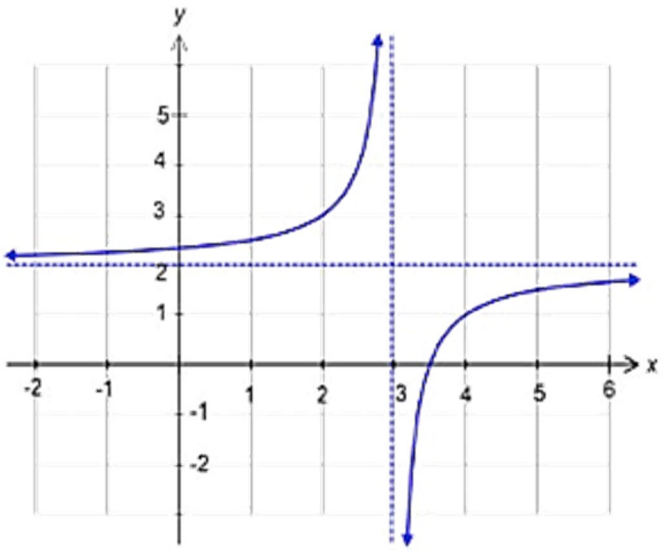
Attempt Questions in the space provided.

Allow about 2 hours and 45 minutes for this section

Part A (14 marks)

Question 11 (3marks)

Part of the function $y = \frac{a}{x+b} + c$ is shown



Find the values of a, b, and c. **3**

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Question 12 (3 marks)

Simplify completely $\frac{ax + ay + 2bx + 2by}{ax - 2ay + 2bx - 4by} \div \frac{xy + y^2}{x^2 - 4xy + 4y^2}$ **3**

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Question 13 (5 marks)

For the function $f(x) = -3cos\left(\frac{\pi x}{2}\right)$,

a. Find the amplitude and period. **2**

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b. Sketch the curve over the domain $\{0 < x < 2\pi\}$. **3**

Question 14 (3 marks)

Let $f(x) = x - 1$ and $g(x) = x^2 + x - 2$. Find :

a. $f(x)g(x)$ **1**

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b. $g(f(x))$ **2**

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End of Part A

NESA Number: | | | | | | | | |

Part B (16 marks)

Question 15 (2 marks)

The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point (-2,0).

What are the values of *m* and *b*? **2**

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Question 16 (3 marks)

Two angles of an isosceles triangle have sizes 150° and 15°. The area of the triangle is 9 cm^2. Find the lengths of all of the sides of the triangle. (Rounded off to 1 decimal point if necessary) **3**

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Question 17 (2 marks)

Complete the square to write the solution to $x^2 + 6x = 1$ in exact form. **2**

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Question 18 (3 marks)

Solve for x and y if $x + 2y = -8$ and $xy = 8$ **3**

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Question 19 (3 marks)

Two hundred tickets are to be sold in a raffle. Two different tickets are to be drawn and are awarded first and second prize respectively. A man buys ten tickets.

Find the probability that he wins at least one prize. **3**

Question 20 (3 marks)

The function $P(x) = x^3 + bx^2 + cx + d$, has *x* intercepts at 3, 0, and -3.

Find *b, c and d.* **3**

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Extra Working Space:

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End of Part B.

NESA Number: | | | | | | | | |

Part C (16 marks)

Question 21 (3 marks)

What is the equation of the normal to the curve $y = x^2 - 4x$ at (1, -3)? **3**

Write your answer in general form.

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Question 22 (2 marks)

If $sin\theta = \frac{5}{13}$ and $cos\theta < 0$, what is the exact value of $tan\theta$? **2**

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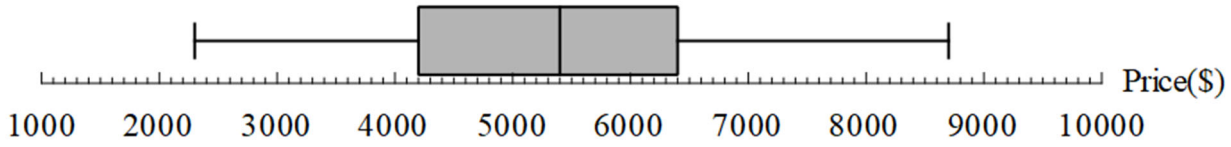
Question 23 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



a. Find the interquartile range. **1**

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b. Show there are no outliers. **2**

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Question 23 continued

c. It is given that the mean is \$5500 dollars. Four paintings of respective prices \$3000, \$3500, \$5900 and \$6800 are now donated to an art gallery. Find the mean of the prices of the remaining paintings in the warehouse. **2**

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Question 24 (5 marks)

The positive numbers $3, k, \frac{3}{2},$ form the start of a geometric sequence.

a. Show that $k = \frac{3}{\sqrt{2}}$. **1**

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Question 24 continued.

b. Find the 9th term of this sequence. **1**

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c. Find S_∞, expressing your answer in the form $p+\sqrt{q}$ where p and q are integers. **3**

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End of Part C.

NESA Number: | | | | | | | | |

Part D (14 marks)

Question 25 (2 marks)

Differentiate. $f(x) = 6x\ tan\left(\frac{1}{x}\right)$ **2**

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Question 26 (2 marks)

Show that $\int_{\ln 2}^{2\ln 2} e^{2x}\,dx = 6$. **2**

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Question 27 (4 marks)

The amount of caffeine, C(t), in milligrams in your system after drinking a cappuccino is given by $C(t) = 105e^{-kt}$ where k is a constant and t is the time in hours that have passed since drinking the cappuccino.

a. After an hour the caffeine in your system has decreased by 30%. Find the exact value of k. **2**

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b. After how many minutes will there be 10 milligrams of caffeine remaining in your system? Give the answer correct to the closest minute. **2**

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Question 28 (3 marks)

A rational function $f(x)$ has the following properties,

- The horizontal asymptote of its graph is $y = 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivates at various points.

	$x < -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x > 2$
$f(x)$			-1		
$f'(x)$	> 0	> 0	0	< 0	< 0
$f''(x)$	> 0	< 0	< 0	< 0	> 0

Sketch $y = f(x)$, using the properties in the table above. **3**

Question 29 (3marks)

A cubical block of ice has an edge of 10 cm. It melts so that its volume decreases at a constant rate of 25 cm^3 per hour and the block remains cubical.

a. Find the volume V at any time t. **2**

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b. What is the time required to completely melt the ice ? **1**

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End of Part D.

NESA Number: | | | | | | | | |

Part E (15 marks)

Question 30 (3 marks)

Find the turning points and points of inflexion for the function $y = x^4 - 2x^3 + 1$ **3**

Question 31 (3 marks)

a. Find $\frac{d}{dx}(xe^x + e^x)$. **1**

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b. Hence find $\int xe^x\,dx$. **2**

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Question 32 (3 marks)

Use the trapezoidal rule with four sub-intervals to evaluate $A = \int_1^5 \frac{7}{x+1}dx$, correct to two decimal places. **3**

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Question 33 (3 marks)

a. Show that $\dfrac{\cos^2\theta}{1-\sin\theta}-\dfrac{\cos^2\theta}{1+\sin\theta}=2\sin x$. **2**

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b. Hence solve $\dfrac{\cos^2\theta}{1-\sin\theta}-\dfrac{\cos^2\theta}{1+\sin\theta}=1$ for $0\le\theta\le\frac{\pi}{2}$. **1**

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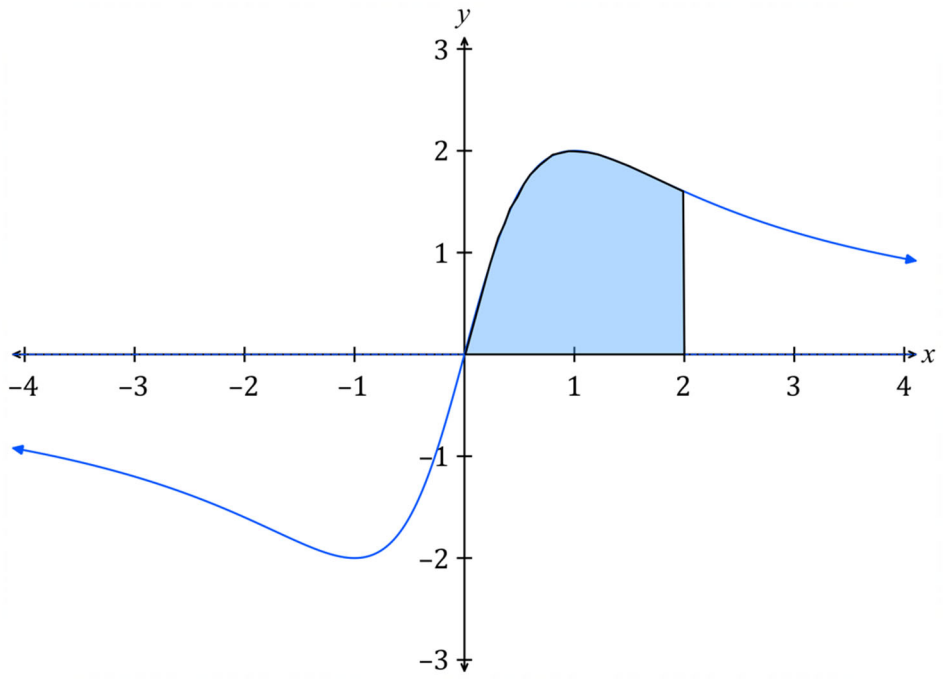
Question 34 (3 marks)

The diagram below shows the graph of $y = \frac{4x}{x^2+1}$.



The region enclosed by the graph, the x-axis and the line x = 2 is shaded.
Calculate the exact value of the area of the shaded region. **3**

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End of Part E.

NESA Number: | | | | | | | | |

Part F (16 marks)

Question 35 (3 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X = x)$	k^2	0.16	0.18	0.3	k	0.12

a. Show that $k = 0.2$. **2**

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b. Calculate $E(X)$. **1**

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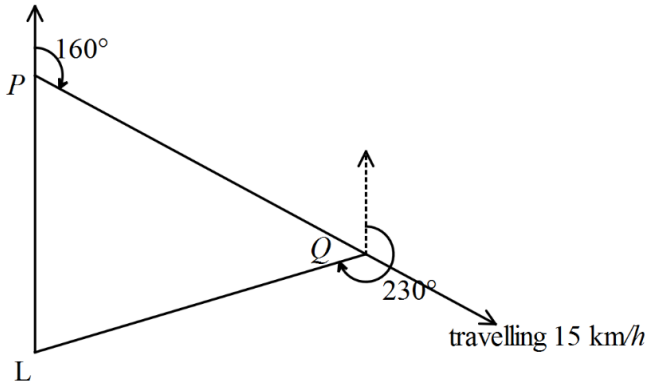
Question 36 (3 marks)

A ship is sailing at 15 km/h on a bearing of $160^\circ T$. At 9:00 am the ship is at P, and the lighthouse, L is due south of the ship. At 9:40 am the ship is at Q, and the lighthouse is on a bearing of $230^\circ T$.



a. Show that $\angle PQL = 110°$ **1**

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b. Find the distance PL. (to 2 decimal places) **2**

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c. Find the time, correct to the nearest minute, at which the lighthouse will be due west of the ship. **3**

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Question 37 (3 marks)

A rectangular prism has a height h cm, width x cm and length $3x$ cm. The surface area of the rectangular prism is 6400 cm^2.

a. Find an expression for h in terms of x. **2**

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b. Show that the volume V cm^3 of the rectangular prism is given by: **1**

$$V = \frac{3x(3200 - 3x^2)}{4}$$

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c. If the volume of the rectangular prism is to be maximised, find the exact values of x and h. **4**

End of Part F.

2023 Mathematics Advanced HSC Trial Examination Solutions

Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1.	A ◉	B ○	C ○	D ○
2.	A ○	B ○	C ◉	D ○
3.	A ○	B ○	C ○	D ◉
4.	A ○	B ◉	C ○	D ○
5.	A ◉	B ○	C ○	D ○
6.	A ◉	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ◉
8.	A ○	B ◉	C ○	D ○
9.	A ○	B ○	C ◉	D ○
10.	A ◉	B ○	C ○	D ○

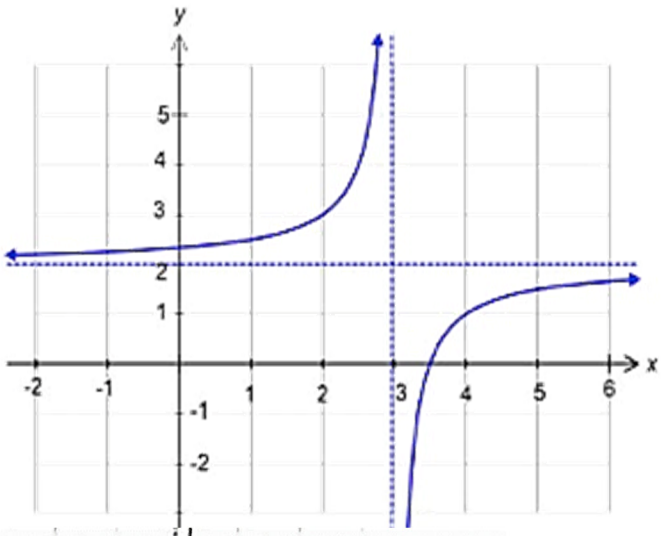
Section II – 90 marks

Part A (14 marks)

Question 11 (3marks)

Part of the function $y = \frac{a}{x+b} + c$ is shown

Find the values of a, b, and c. **3**



$a = -1$ ✓

$b = -3$

$c = 2$ ✓

Pt (4, 1)

$1 = \frac{a}{4-3} + 2$ ✓

$1 = a + 2$

Markers Comments:
1 mark for getting each a, b, c
many students did not substitute correctly to in to find the value of a

Question 12 (3 marks)

Simplify completely $\dfrac{ax+ay+2bx+2by}{ax-2ay+2bx-4by} \div \dfrac{xy+y^2}{x^2-4xy+4y^2}$ **3**

$$\frac{a(x+y)+2b(x+y)}{a(x-2y)+2b(x-2y)} \times \frac{(x-2y)(x-2y)}{y(x+y)}$$

$$= \frac{(a+2b)(x+y)(x-2y)(x-2y)}{(a+2b)(x-2y)y(x+y)}$$ ✓ - 2 factorisations ✓✓ - all 4 factorisations.

$$= \frac{x-2y}{y}$$ ✓

Markers Comments:
Poorly done.
Students need to clearly factorised both fractions (1 mark each) and cancel (1 mark)
Students who cancelled without factorising first did not get the mark

Question 13 (5 marks)

For the function $f(x) = -3\cos\left(\frac{\pi x}{2}\right)$,

a. Find the amplitude and period. **2**

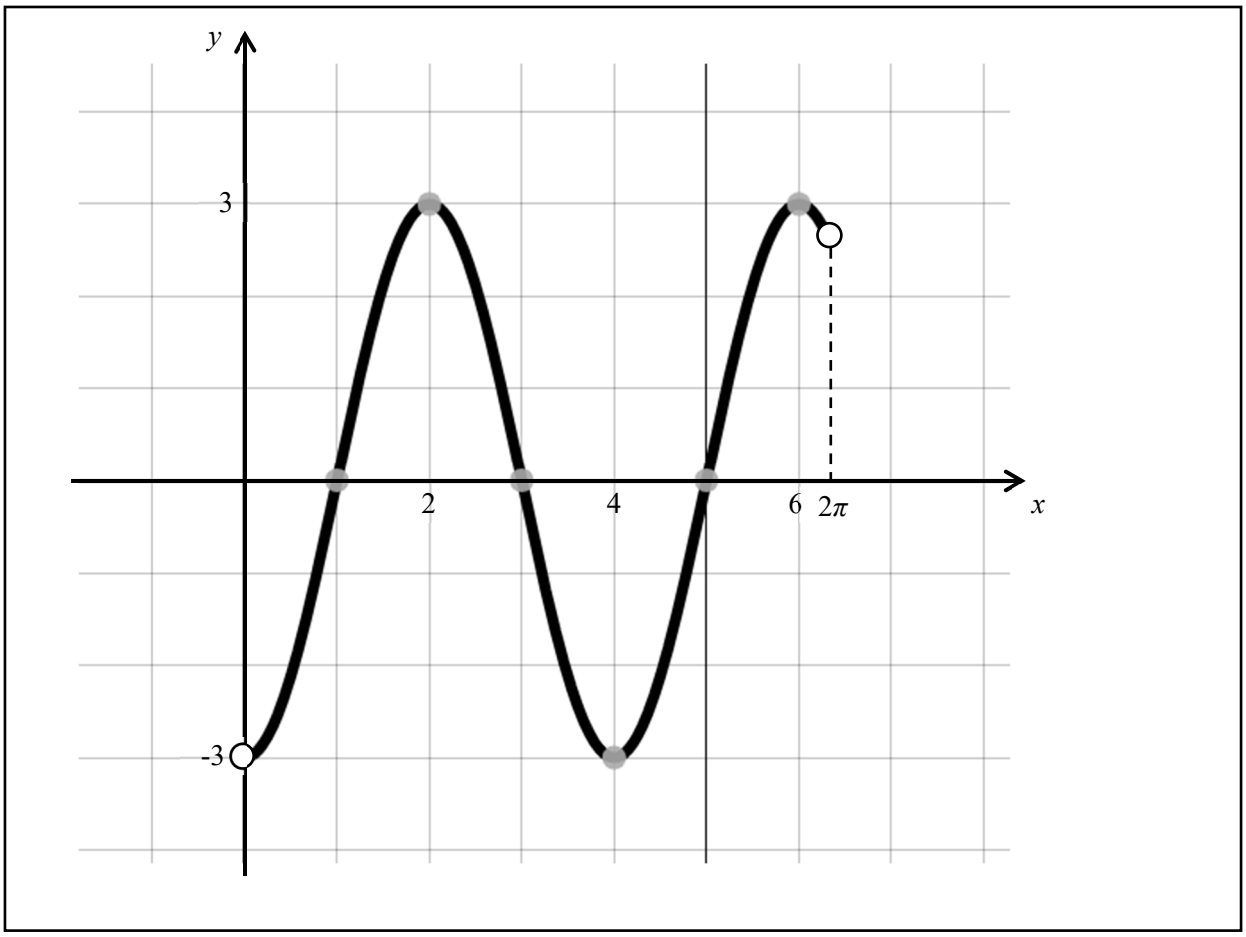
Amplitude = 3 ✓

Period = $\frac{2\pi}{n} = \frac{2\pi}{\pi/2} = 4$ ✓

Markers Comments:
The amplitude (3) must be positive.

b. Sketch the curve over the domain $\{0 < x < 2\pi\}$. **3**

c.



Markers Comments:
Poorly done.
1 mark for the correct period (approximately 1.5 periods))
1 mark for drawing a correct cos graph with amplitude of 3, starting at -1
1 mark for having the majority of the following features of a graph:
labelled point, title, labelled and correctly scaled axis, open circles at the end points, and a neatly drawn sketch of a cos curve, with curves not straight lines

period ✓
amplitude = 3 ✓
end points ✓

Question 14 (3 marks)

Let $f(x) = x - 1$ and $g(x) = x^2 + x - 2$. Find :

a. $f(x)g(x)$ **1**

$(x-1)(x^2+x-2)$

$= x^3 + x^2 - 2x - x^2 - x + 2$

$= x^3 - 3x + 2$ ✓

Markers Comments:
Well done.

b. $g(f(x))$ **2**

$g(x-1) = (x-1)^2 + (x-1) - 2$ ✓

$= x^2 - 2x + 1 + x - 1 - 2$

$= x^2 - x - 2$ ✓

Markers Comments:
Well done.

End of Part A

Part B (16 marks)

Question 15 (2 marks)

The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point (-2,0).

What are the values of *m* and *b*? **2**

$y' = 3x^2 - 3$

When $x = -2$, $m = 9$ ✓

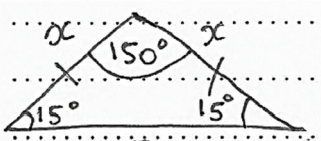
$0 = 9 \times -2 + b$

$\therefore b = 18$ ✓

Markers Comments:
Mostly answered well.

Question 16 (3 marks)

Two angles of an isosceles triangle have sizes 150° and 15°. The area of the triangle is 9 cm^2. Find the lengths of all of the sides of the triangle. (Rounded off to 1 decimal point if necessary) **3**



$9 = \frac{1}{2} x^2 \sin 150°$ ✓

$18 = x^2 \times \frac{1}{2}$

$36 = x^2$

$\therefore x = 6$ ✓ $(x \neq -6)$

$\frac{y/2}{6} = \cos 15°$

$\frac{y}{2} = 6 \cos 15°$

$y = 12 \cos 15° = 11.6$ ✓

Markers Comments:
No mark given if angle of 15 was used for finding the area instead of 150. Students were more successful when they included a diagram of the information given showing two equal sides including the angle of 150.

Question 17 (2 marks)

Complete the square to write the solution to $x^2 + 6x = 1$ in exact form. **2**

$x^2 + 6x = 1$

$x^2 + 6x + 9 = 10$

$(x+3)^2 = 10$ ✓

$x + 3 = \pm\sqrt{10}$

$x = -3 \pm \sqrt{10}$ ✓

Markers Comments:
Mostly answered well. Many students failed to give two solutions as they did not take both positive and negative square root of 10.

Question 18 (3 marks)

Solve for x and y if $x + 2y = -8$ and $xy = 8$ **3**

$x + 2y = -8$ (1) $\quad y = \frac{8}{x}$ (2)

Sub (2) into (1)

$x + \frac{2 \times 8}{x} = -8$

$x + \frac{16}{x} = -8$

$x^2 + 16 = -8x$

$x^2 + 8x + 16 = 0$

$(x+4)^2 = 0$

$\therefore x = -4$

when $x = -4$

$y = \frac{8}{-4}$

$\therefore y = -2$

Markers Comments:
Mostly well done, a few simply careless algebraic errors making the quadratic difficult to solve. A check step is needed to pick up on simple errors.

Question 19 (3 marks)

Two hundred tickets are to be sold in a raffle. Two different tickets are to be drawn and are awarded first and second prize respectively. A man buys ten tickets.

Find the probability that he wins at least one prize. **3**

$P(\bar{F}S)$ or $P(F\bar{S})$ or $P(FS)$

$P(\bar{F}S) = \left(1 - \frac{10}{200}\right) \times \frac{10}{199}$

$= \frac{19}{398}$

$P(F\bar{S}) = \frac{10}{200} \times \frac{190}{199}$

$= \frac{19}{398}$

2 probabilities ✓

3 probabilities ✓✓

$P(FS) = \frac{10}{200} \times \frac{9}{199}$

$= \frac{9}{3980}$

$\frac{19}{398} + \frac{19}{398} + \frac{9}{3980}$

$= \frac{389}{3980}$ ✓

Markers Comments:
Well done.

Question 20 (3 marks)

The function $P(x) = x^3 + bx^2 + cx + d$, has *x* intercepts at 3, 0, and -3.

Find *b, c and d.* **3**

$P(x) = x(x-3)(x+3)$ ✓

$= x(x^2 - 9)$

$= x^3 - 9x$

$= x^3 + 0x^2 - 9x + 0$

$\therefore b = 0$

$c = -9$

$d = 0$

✓✓ three correct.

(-1) for each mistake.

Markers Comments:

End of Part B.

Part C (16 marks)

Question 21 (3 marks)

What is the equation of the normal to the curve $y = x^2 - 4x$ at (1, -3)? **3**

Write your answer in general form.

$y' = 2x - 4 = m$ ✓

when $x = 1$, $m_1 = -2$

gradient of normal $= \frac{1}{2}$ ✓ $(m_1 m_2 = -1)$

$y + 3 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x - \frac{1}{2} - 3$

$y = \frac{1}{2}x - \frac{7}{2}$

$2y = x - 7$ (-1 if not in general form.)

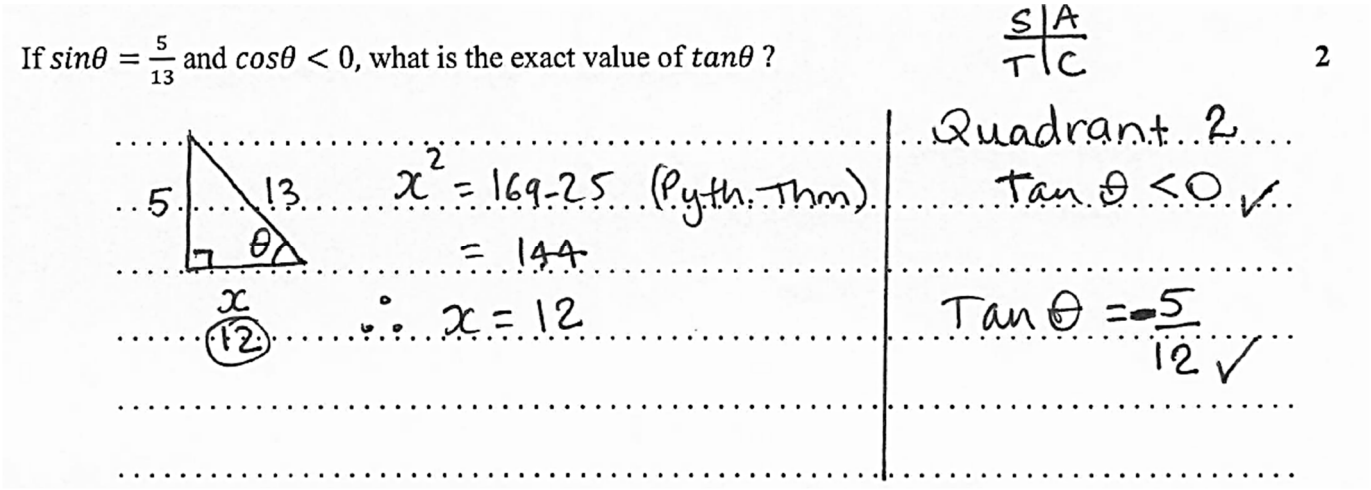
$x - 2y - 7 = 0$ ✓

Markers Comments:
A large number of students couldn't find the correct gradient of either tangent or normal.

Some students did not give their answer in general form, resulting in loss of one mark.

Question 22 (2 marks)

If $sin\theta = \frac{5}{13}$ and $cos\theta < 0$, what is the exact value of $tan\theta$? **2**

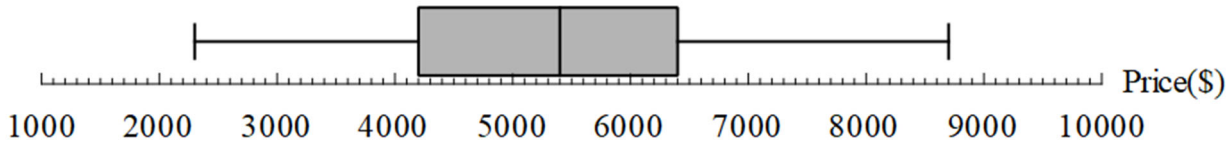


Markers Comments:

Majority of the students got the correct value for $\tan\theta = \frac{5}{12}$ but they did not realise that θ is in the second quadrant. Therefore $\tan\theta < 0$.

Question 23 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



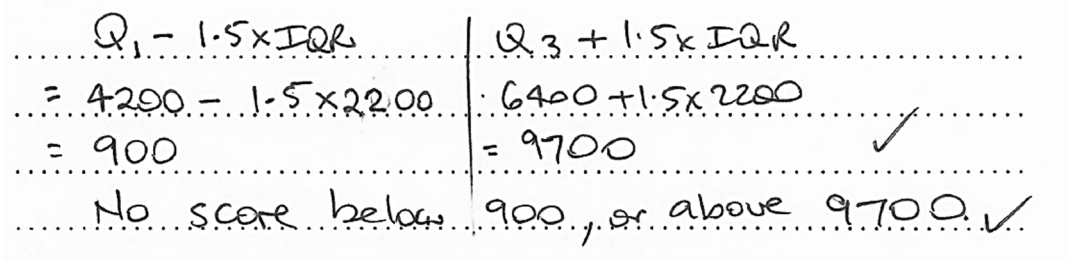
a. Find the interquartile range. **1**

$IQR = 6400 - 4200$
$= 2200$

Markers Comments:

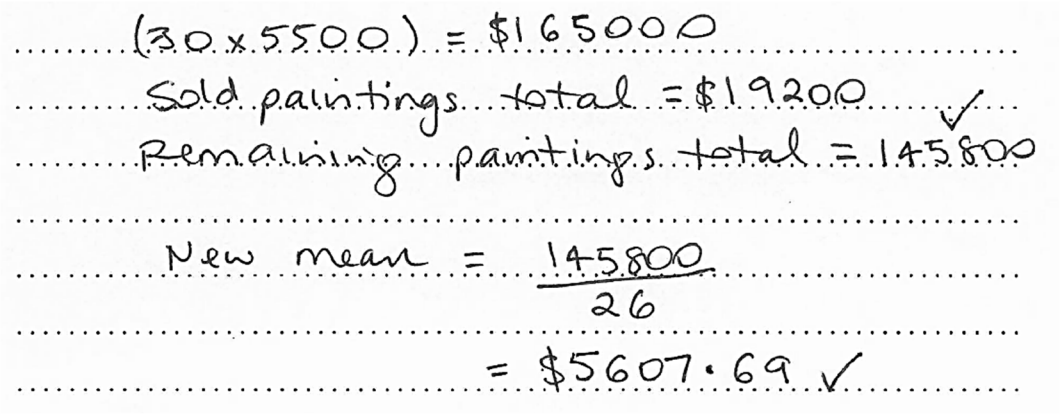
Well done.

b. Show there are no outliers. **2**



Markers Comments:
A large number of students couldn't demonstrate the clear understanding of outliers. They are advised to revise this and look at the solutions provided here.

c. It is given that the mean is \$5500 dollars. Four paintings of respective prices \$3000, \$3500, \$5900 and \$6800 are now donated to an art gallery. Find the mean of the prices of the remaining paintings in the warehouse. **2**

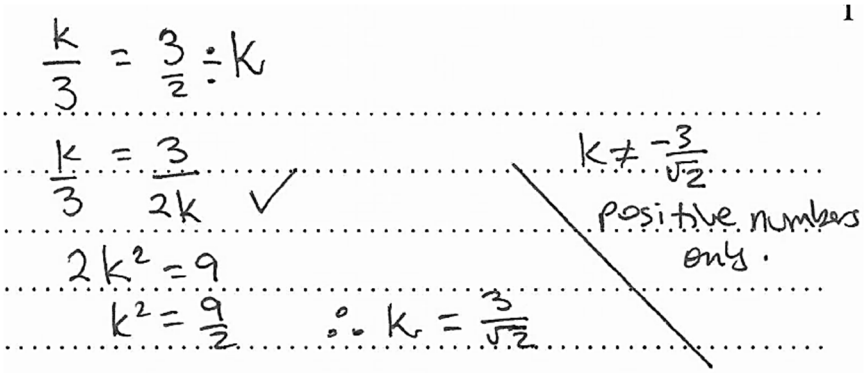


Markers Comments:
Overall, well done.
Some students did not read the question carefully and instead of taking out the prices of the denoted paintings, they added their prices to the original price and used the wrong number of paintings to fimd out the new mean..

Question 24 (5 marks)

The positive numbers $3, k, \frac{3}{2}$, form the start of a geometric sequence.

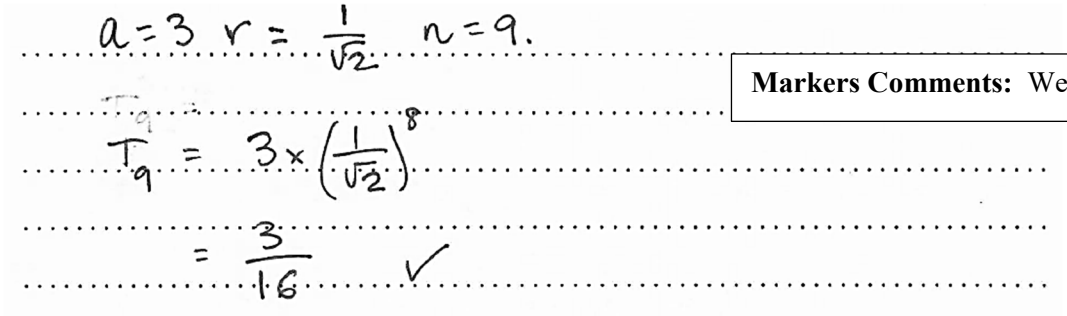
a. Show that $k = \frac{3}{\sqrt{2}}$. **1**



Markers Comments:

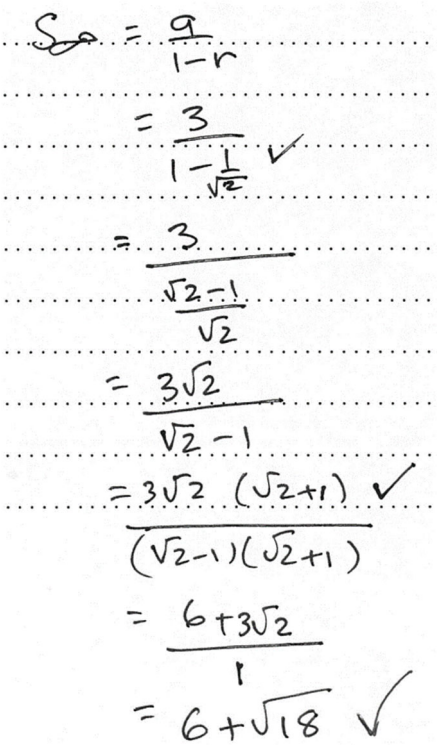
Students are reminded that the question was asking to prove that $k = \frac{3}{\sqrt{2}}$. A large number of students used $k = \frac{3}{\sqrt{2}}$ to find out r and then use r to prove that $k = \frac{3}{\sqrt{2}}$.

b. Find the 9th term of this sequence. **1**



Markers Comments: Well done.

c. Find S_∞, expressing your answer in the form $p + \sqrt{q}$ where p and q are integers. **3**

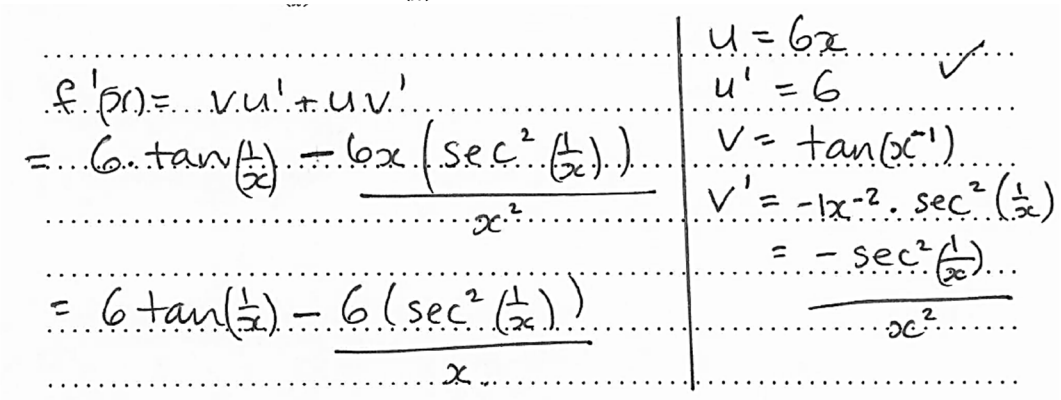


Markers Comments:
Overall, well done. Some students lost one mark as they did not give their answer in the required form.

Part D (14 marks)

Question 25 (2 marks)

Differentiate. $f(x) = 6x\ tan\left(\frac{1}{x}\right)$ **2**

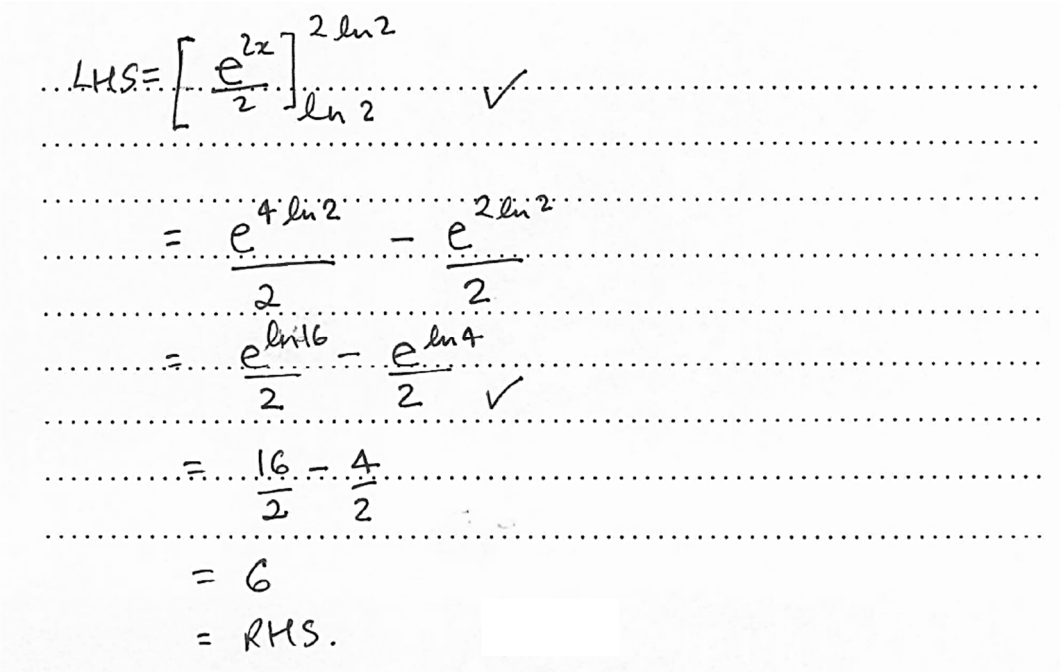


Markers Comments:

Answered well.

Question 26 (2 marks)

Show that $\int_{\ln 2}^{2\ln 2} e^{2x}\,dx = 6$. **2**



Markers Comments:

Most students achieved full marks. Those who did not, lost one mark for not showing enough working/reasoning to demonstrate their understanding. Students are reminded that for show questions, they must produce full working out.

Question 27 (4 marks)

The amount of caffeine, C(t), in milligrams in your system after drinking a cappuccino is given by $C(t) = 105e^{-kt}$ where k is a constant and t is the time in hours that have passed since drinking the cappuccino.

a. After an hour the caffeine in your system has decreased by 30%. Find the exact value of k. **2**

$0.7 = e^{-k \times 1}$ ✓

$k = -\ln 0.7$ ✓

(~~$0.7 \times 105 = 105 e^{-kt}$~~ $0.7 = e^{-k \times 1}$)

Markers Comments:
Generally answered well. Common errors were using 30% in place of 70% and not working out 70% of 105.

b. After how many minutes will there be 10 milligrams of caffeine remaining in your system? Give the answer correct to the closest minute. **2**

$10 = 105 e^{-kt}$

$\frac{10}{105} = e^{-kt}$

$\ln\left(\frac{2}{21}\right) = -kt$ ✓

$\therefore t = \ln\left(\frac{2}{21}\right) \div -\ln(0.7)$

$= 6.592$ hrs

$= 396$ min ✓

Markers Comments:

Generally answered well. There are some students who need to revise their work with Logs and Exponentials.

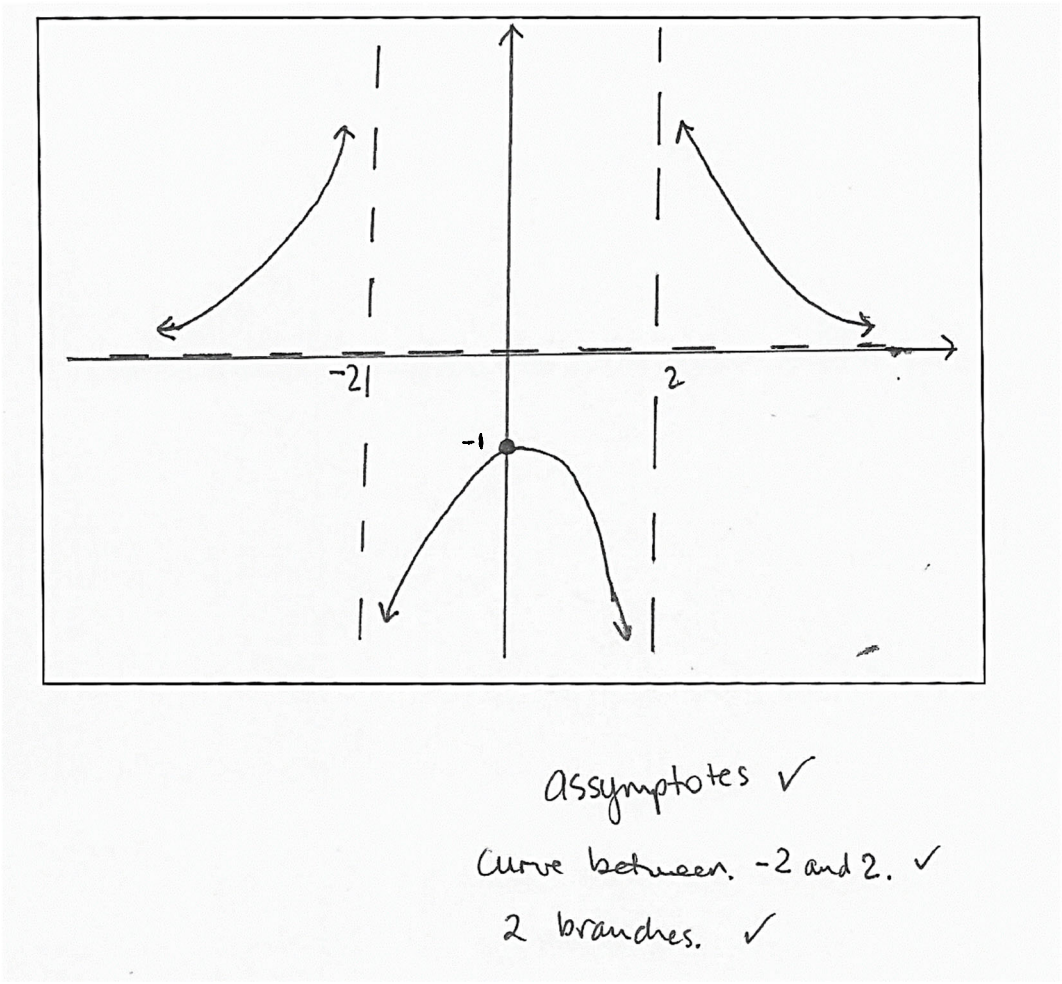
Question 28 (3 marks)

A rational function $f(x)$ has the following properties,

- The horizontal asymptote of its graph is $y = 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivates at various points.

	$x < -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x > 2$
$f(x)$			-1		
$f'(x)$	> 0	> 0	0	< 0	< 0
$f''(x)$	> 0	< 0	< 0	< 0	> 0

Sketch $y = f(x)$, using the properties in the table above. **3**



Markers Comments:
Although generally answered well, many students lost a mark for not clearly indicating the horizontal asymptote.

Question 29 (3marks)

A cubical block of ice has an edge of 10 cm. It melts so that its volume decreases at a constant rate of 25 cm^3 per hour and the block remains cubical.

a. Find the volume V at any time t. **2**

$\frac{dV}{dt} = -25$ ✓

$V = -25t + c$

when $t = 0$, $V = 1000$

$1000 = 0 + c$

$\therefore c = 1000$

$V = -25t + 1000$ ✓

Markers Comments:
Most students did not use the strategy shown here. To achieve full marks, students needed to show some working. If they just produced the answer, they were awarded only 1 mark.

b. What is the time required to completely melt the ice ? **1**

$0 = -25t + 1000$

$25t = 1000$

$\therefore t = 40$ hours ✓

Markers Comments:

Answered well.

End of Part D.

Part E (15 marks)

Question 30 (3 marks)

Find the turning points and points of inflexion for the function $y = x^4 - 2x^3 + 1$ **3**

$y = x^4 - 2x^3 + 1$

$y' = 4x^3 - 6x^2$

$= 2x^2(2x - 3)$

$y'' = 12x^2 - 12x$

$= 12x(x - 1)$ S.P. at $(0,1)$ ✓ $(1.5, -\frac{11}{16})$

Turning Points:

when $y' = 0$, $x = 0, \frac{3}{2}$

x	-1	0	1	$3/2$	2
y'	-ve (−10)	0	-ve (−2)	0	+ve (8)

Turning point at $(1.5, -\frac{11}{16})$ ✓

[$(1.5, -\frac{11}{16})$ is a minimum turning point.] (Must have test)

Points of inflexion:

when $y'' = 0$ $x = 0, 1$

x	-1	0	0.5	1	2
y''	+ve (24)	0	-ve (−3)	0	+ve (24)

∴ Points of inflexion at

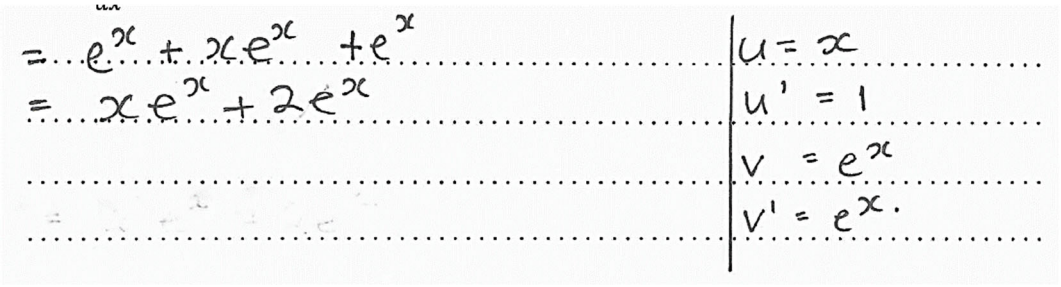
$(0, 1)$ $(1, 0)$ ✓

$(0,1)$ is HORIZ. POI. (must have test.)

Markers Comments:
Students need to realise that y'=0 finds stationary points, and to justify a turning point there has to be a test to show change of gradient. The test to show change of concavity for a point of inflexion must also be included.

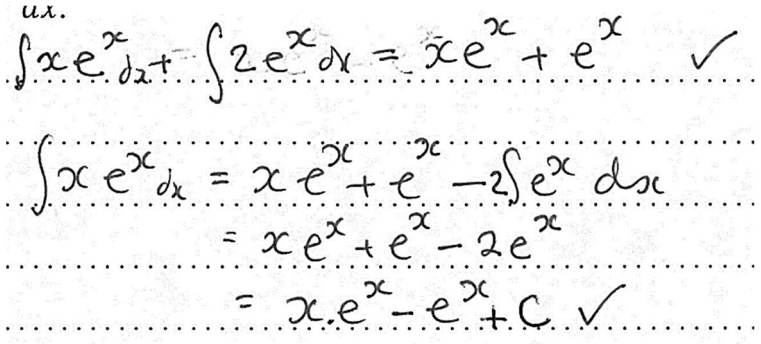
Question 31 (3 marks)

a. Find $\frac{d}{dx}(xe^x + e^x)$. **1**



Markers Comments:
Well done.

b. Hence find $\int xe^x\,dx$. **2**



Markers Comments:
Students were sometimes confused and didn't know how to integrate (and manipulate) the result from part a.

Question 32 (3 marks)

Use the trapezoidal rule with four sub-intervals to evaluate $A = \int_1^5 \frac{7}{x+1}\,dx$, correct to two decimal places. **3**

x	1	2	3	4	5
$f(x)$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$

Area $= \frac{1}{2}\left(\frac{7}{2} + 2\left(\frac{7}{3}+\frac{7}{4}+\frac{7}{5}\right) + \frac{7}{6}\right)$ ✓

$= 7.82\ u^2$. ✓

Markers Comments:
Well done except for some students who had the wrong number of sub-intervals.

Question 33 (3 marks)

a. Show that $\frac{\cos^2\theta}{1-\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta} = 2\sin x$. **2**

$\frac{\cos^2\theta(1+\sin\theta) - \cos^2\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos^2\theta + \cos^2\theta\sin\theta - \cos^2\theta + \sin\theta\cos^2\theta}{1-\sin^2\theta}$

$= \frac{2\cos^2\theta\sin\theta}{\cos^2\theta}$

$= 2\sin\theta$

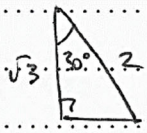
Markers Comments:
Well done. Multiple solutions possible.

b. Hence solve $\dfrac{\cos^2\theta}{1-\sin\theta}-\dfrac{\cos^2\theta}{1+\sin\theta}=1$ for $0 \le \theta \le \dfrac{\pi}{2}$. **1**

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$ ✓

$\theta = \frac{\pi}{6}$ ✓

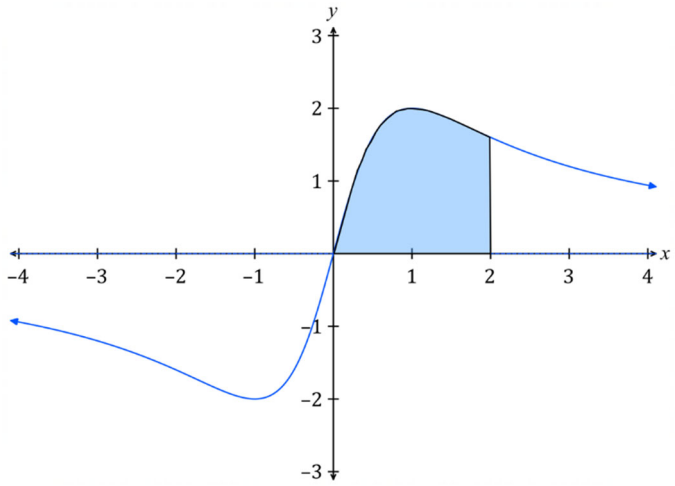


Markers Comments:
Well done. Some students ignored the given domain.

Question 34 (3 marks)

The diagram below shows the graph of $y = \dfrac{4x}{x^2+1}$.

The region enclosed by the graph, the x-axis and the line x = 2 is shaded.
Calculate the exact value of the area of the shaded region. **3**



$\int_0^2 \dfrac{4x}{x^2+1}\,dx = 2\int_0^2 \dfrac{2x}{x^2+1}\,dx$ ✓

$= 2\left[\ln(x^2+1)\right]_0^2$ ✓

$= 2(\ln 5 - \ln 1)$

$= 2\ln 5 \text{ u}^2$ ✓

Markers Comments:
Well done.

End of Part E.

Part F (16 marks)

Question 35 (3 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X=x)$	k^2	0.16	0.18	0.3	k	0.12

a. Show that $k = 0.2$. **2**

$k^2 + 0.16 + 0.18 + 0.3 + k + 0.12 = 1$

$k^2 + k + 0.76 = 1$

$k^2 + k - 0.24 = 0$ ✓

$k = \frac{-1 \pm \sqrt{1 + 4 \times 1 \times 0.24}}{2}$

$= \frac{-1 \pm \sqrt{1.96}}{2} = \frac{-1 \pm 1.4}{2} = 0.2, -1.2$

$k \neq -1.2$ ✓

Markers Comments:
Mostly well done. All the students are able to obtain the quadratic equation. However, several students attempt showing $k = 0.2$ by substituting $k = 0.2$ into the equation or solving the equation based on the assumption of $k = 0.2$ which leads to one mark deduction. To be able to get the second mark, students need to show clear working out of solving the quadratic equation and provide the correct answer.

b. Calculate $E(X)$. **1**

$(0 \times 0.04) + (2 \times 0.16) + (4 \times 0.18) + (5 \times 0.3)$

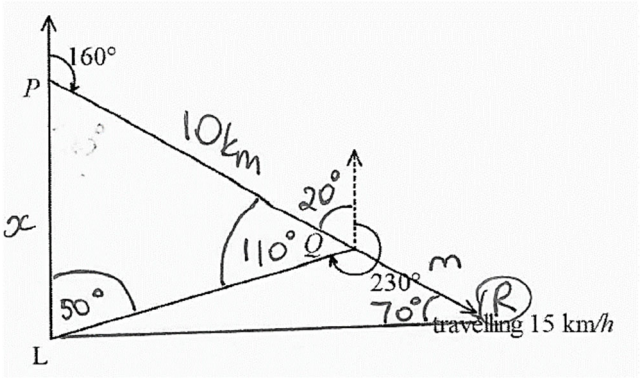
$+ (8 \times 0.2) + (9 \times 0.12)$

$= 5.22$ ✓

Markers Comments:
Answered well by most student.

Question 36 (3 marks)

A ship is sailing at 15 km/h on a bearing of $160^\circ T$. At 9:00 am the ship is at P, and the lighthouse, L is due south of the ship. At 9:40 am the ship is at Q, and the lighthouse is on a bearing of $230^\circ T$.



a. Show that $\angle PQL = 110°$ **1**

$\angle PQL = 360° - (230+20)$ ✓(angles at a point)
$= 110°$
$(20° =$ cointerior angles)

Markers Comments:
Answered well by most student. Students know how to use co-interior angles to achieve the result.

b. Find the distance PL. (to 2 decimal places) **2**

let $PL = x$ $PQ = \frac{40}{60} \times 15 = 10$ km.

$\frac{x}{\sin 110} = \frac{10}{\sin 50}$

$x = \frac{10 \times \sin 110}{\sin 50}$

$= 12.27$ km. ✓

Markers Comments:
Answered well by majority of the students. A couple of students made calculation error in the final step. A few students did not round up to 12.27km and lost one mark.

c. Find the time, correct to the nearest minute, at which the lighthouse will be due west of the ship. **3**

Let end position be R. Let LR = y
Let QR = m
$\sin 70^\circ = \frac{12.27}{10+m}$ ✓
$10+m = \frac{12.27}{\sin 70^\circ}$
$m = \frac{12.27}{\sin 70} - 10$
$= 3.05746 \text{ km}$.

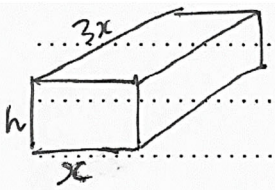
Time to travel 'm'
$\frac{3.05746}{15} \times 60$
$= 12 \text{ min}$. ✓
∴ 9.52am.

Markers Comments:
Most students achieved at least one mark for this part. Some students have misinterpreted the question, they spent a lot of effort finding QL. Many students did not provide the final answer using the exact time: 9.52am as requested by the question, one mark is lost due to this reason. Carry-over mark is awarded if students made a calculation error from part b and did not make the following part easier.

Question 37 (3 marks)

A rectangular prism has a height h cm, width x cm and length $3x$ cm. The surface area of the rectangular prism is 6400 cm^2.

a. Find an expression for h in terms of x. **2**



$2hx + 6x^2 + 6xh = 6400$
$8hx + 6x^2 = 6400$
$8hx = 6400 - 6x^2$
$h = \frac{6400}{8x} - \frac{6x^2}{8x} = \frac{6400-6x^2}{8x}$
$= \frac{3200-3x^2}{4x}$ ✓ 1

Markers Comments:
Most students answered this part well.

A couple of students did not provide a correct formula for finding the surface area of the rectangular prism and did not get the marks.

b. Show that the volume V cm^3 of the rectangular prism is given by: **1**

$$V = \frac{3x(3200 - 3x^2)}{4}$$

$V = 3x \times h \times x$

$= 3x^2 h$

$= 3x^2 \left(\frac{3200 - 3x^2}{4x}\right)$ ✓

$= \frac{3x(3200 - 3x^2)}{4}$

$= \frac{9600x}{4} - \frac{9x^3}{4} = 2400x - \frac{9}{4}x^3$

Markers Comments:
Most students demonstrate the understanding of volume of prism and achieved one mark.

Carry-over mark will NOT be awarded if the students used the wrong expression from part a) but reached the correct expression for this part.

c. If the volume of the rectangular prism is to be maximised, find the exact values of x and h. **4**

$\frac{dV}{dx} = 2400 - \frac{27}{4}x^2$

$0 = 2400 - \frac{27}{4}x^2$ ✓

$\frac{27}{4}x^2 = 2400$

$x^2 = \frac{9600}{27}$

$x^2 = \frac{9600}{27}$ ✓

$x = \frac{\sqrt{3200}}{3}$

$\therefore x = \frac{40\sqrt{2}}{3}$ ✓

Test for max. TP

x	10	$40\sqrt{2}/3$	20
$\frac{dV}{dx}$	1725	0	-300

$h = 3200 - 3 \times \left(\frac{40\sqrt{2}}{3}\right)^2$

$= 3200 - \frac{4 \times 40\sqrt{2}}{3}$

$= 3200 - \frac{3200}{3}$

$= \frac{160\sqrt{2}}{3}$

$= \frac{6400}{160\sqrt{2}}$

$h = \frac{40}{\sqrt{2}}$ ✓

Markers Comments:
Most students achieved at least one mark for this part as students demonstrate understanding of finding the first derivative for the volume and set which equals to zero in order to find its maximum volume. Students must make sure they have included the test for maximum point, otherwise they will lose one mark. Because the purpose of this part is not assessing one's ability to simplify expression, marks are not deducted if the answer is not in its simplest form.

End of Part F.